

Analysis the neutral triplet collective mode in honeycomb lattice: without one-cone approximation

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In this research has been studied the possibility of existence the neutral triplet collective mode in undoped graphene and graphite without one-cone approximation. This work tries to study this collective mode from different points of view. It also studies the effects of on-site Hubbard interaction on neutral collective mode in honeycomb lattice via two representations (sub-lattice and eigenvector). The effective long range interaction in honeycomb lattice has been introduced for studying the spin collective mode. This work shows that short and effective long range interactions are not responsible for the formation of neutral triplet collective mode in graphene without *one-cone* approximation.

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INTRODUCTION

The discovery of graphene, one layer of carbon atoms, arose high hopes for its application in future of electronic devices [1]. One reason for this attention is high mobility of charge carriers (electrons or holes) whose density are well controlled by electrical and chemical doping. Although the application of pure graphene in electronic industry didn't develop because of the gap-less dispersion. Adding atoms and doping in graphene *e.g.* fluorinated mono-vacancies [2], Ge-interrelated [3] have been studied for investigation the electrical characteristics of graphene under impurity doping, which gives rise to exploring new interesting electronic and magnetic properties.

The prediction of existence the neutral triplet collective mode in a honeycomb lattice is considered to be the new idea [4]. This collective mode was predicted by considering random phase approximation (RPA) method in Hubbard model on the honeycomb lattice *e.g.* undoped graphene and graphite. By considering this collective mode, good agreement between experimental and theoretical results in TRPES experiment on graphite by Momentum Average (MA) method was achieved [5]. In this approximation, neutral triplet spin-1 mode was considered as coupling between two spinon bound states in undoped graphene but in scalar form not in 2×2 matrix [6], [4]. If we study and recheck the calculations of ref. [6, 7], we will find, first: some parts of the calculations between two sub-lattice are eliminated and second: effects of overlap factors are neglected [8, 9]. The innovation of ref. [6] was considering scalar form for spin susceptibility, instead of 2×2 tensor for two sub-lattice material. The essential purpose of this research is analyzing the validity of spin-1 collective mode without one-cone approximation in Hubbard model and its novelty is calculating the effects of effective long range interaction (U_{l-r}^{eff}) in RPA method on neutral triplet collective mode [10]. Why we considered this form for long range interaction? Because by considering only this interaction type we will obtain Eq. 26 which is an essential equation in finding neutral collective mode in RPA formulas (without one-cone approximation), in addition we should consider those types of interaction, acting between electrons with opposite spins. The effects of Coulomb interaction is known to be in the form $e^2/(\epsilon r)$ which lead to prediction of plasmon dispersion in graphene and has been discussed in details in ref. [11]. The coulomb interaction in the honey comb lattice didn't result spin-1 collective mode.

In the first section of this paper, the effects of short range Hubbard interaction in RPA method on honeycomb lattice was calculated exactly without neglecting any interaction terms [8], in two representations. In the second section, we first introduced new effective long range interaction as a new model in graphene and then we used this model for studying the neutral triplet collective mode. finally the finding of these calculations are summarized and discussed on validation of neutral collective mode in undoped graphene.

ON-SITE INTERACTION: SUB LATTICE REPRESENTATION

The Hubbard Hamiltonian consist of two terms: tight binding (TB) and short range Hubbard interaction (HI),

$$H = H_{TB} + H_{HI} = -t \sum_{\langle i,j \rangle, \sigma} (a_i^{\dagger \sigma} b_j^{\sigma} + b_j^{\dagger \sigma} a_i^{\sigma}) + U \sum_i (a_i^{\dagger \uparrow} a_i^{\uparrow} a_i^{\dagger \downarrow} a_i^{\downarrow} + b_i^{\dagger \uparrow} b_i^{\uparrow} b_i^{\dagger \downarrow} b_i^{\downarrow}), \quad (1)$$

where t is the nearest neighbor hopping. $a_i^{\sigma}, b_i^{\sigma}, (a_i^{\dagger \sigma}, b_i^{\dagger \sigma})$ are annihilation (creation) operators for electrons in sub lattices

A, B respectively, and σ stands for spin of electrons, and U is the Hubbard on-site interaction potential. The Hubbard interaction term can be written in the exchange channel,

$$H_{HI} = -U \sum_i (a_i^{\uparrow\dagger} a_i^{\downarrow} a_i^{\uparrow\dagger} a_i^{\downarrow} + b_i^{\uparrow\dagger} b_i^{\downarrow} b_i^{\uparrow\dagger} b_i^{\downarrow}) \quad (2)$$

The interaction term is represented by Feynman diagrams with two kinds of vertices. Each vertex corresponds to the on-site repulsion interaction on the A, B sub-lattices Fig. 1. In each vertex electrons with opposite spins interact via repulsion potential [12]. Fig.(1-a) shows creation (annihilation) of electrons with up and down spin in each vertex either in A or B sub-lattice. In Fig.(1-b), by adding vertices of each sub-lattice, RPA chain is formed. In this chain the vertex with the same or different sub-lattice put to gather, so we define polarization operators, χ_{AA}, χ_{AB} which include effects of overlap factors.

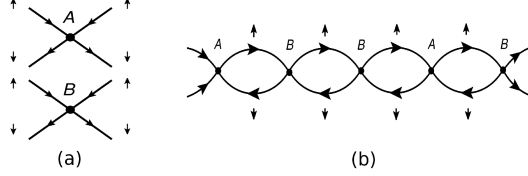


FIG. 1: (a) Each vertex includes creation(destruction) of an electron with up (down) spin. These vertices show interaction between two electrons with opposite spin in same atoms in sub-lattices by short range repulsive potential U . This type of interaction is shown in Eq. 2, in each sub-lattice $A(B)$ electrons with up(down) spin interact to electrons with down(up) spin. (b) The RPA chain includes summing of A, B vertices. This figure is one of the RPA chains.

Each vertex in the chain can be either of A or B type. By summing the RPA series, one can achieve to the collection of equations for susceptibilities in the sub-lattice representation [8],

$$\begin{cases} \chi_{AA} = \chi_{AA}^0 + U\chi_{AA}^0\chi_{AA} + U\chi_{AB}^0\chi_{BA} \\ \chi_{AB} = \chi_{AB}^0 + U\chi_{AA}^0\chi_{AB} + U\chi_{AB}^0\chi_{BB} \\ \chi_{BA} = \chi_{BA}^0 + U\chi_{BA}^0\chi_{AA} + U\chi_{BB}^0\chi_{BA} \\ \chi_{BB} = \chi_{BB}^0 + U\chi_{BA}^0\chi_{AB} + U\chi_{BB}^0\chi_{BB} \end{cases} \quad (3)$$

The coupling equations in Eq. 3 should be decoupled, so giving two subsystems with the common determinant, which should be zero at the plasmon dispersion [8]:

$$\begin{vmatrix} 1 - U\chi_{AA}^0 & -U\chi_{AB}^0 \\ -U\chi_{BA}^0 & 1 - U\chi_{BB}^0 \end{vmatrix} = 0, \quad (4)$$

or

$$1 - U(\chi_{AA}^0 + \chi_{BB}^0) + U^2\chi_{AA}^0\chi_{BB}^0 - U^2\chi_{AB}^0\chi_{BA}^0 = 0. \quad (5)$$

The bare polarization operators or susceptibilities of vertices can be calculated using non-interacting Green's function for graphene's electrons in the sub-lattice representation. By considering the Green's functions in the momentum representation and close to Dirac points, classification of Green's function by electron valleys \vec{K} and $-\vec{K}$, by vicinities to the corresponding Dirac points was achieved. In the valleys \vec{K} and $-\vec{K}$ the non-interacting Hamiltonians in the representation of sub lattice A, B are

$$H_{\pm\vec{K}} = \begin{pmatrix} 0 & \pm p_x - ip_y \\ \pm p_x + ip_y & 0 \end{pmatrix}, \quad (6)$$

here by considering $v_F = 1$, the corresponding Matsubara Green's functions for the undoped graphene ($\mu = 0$) are

$$G^{\pm\vec{K}}(\vec{p}, i\epsilon) = \frac{1}{(i\epsilon)^2 - p^2} \begin{pmatrix} i\epsilon & \pm p e^{\mp i\phi} \\ \pm p e^{\pm i\phi} & i\epsilon \end{pmatrix}, \quad (7)$$

where ϕ is the azimuthal angle of the momentum vector \vec{p} . By using Eq. 7, the polarization operators can be calculated as

$$\chi_{\alpha,\beta}^0(\vec{q}, i\omega) = -T \sum_{v=\pm\vec{K}} \sum_{\epsilon} \int \frac{d\vec{p}}{(2\pi)^2} G_{\alpha,\beta}^v(\vec{p}, i\epsilon) G_{\beta,\alpha}^{-v}(\vec{p}', i\epsilon + i\omega), \quad (8)$$

where $\vec{p}' = \vec{p} + \vec{q}$. By performing the summation over fermionic Matsubara frequencies $\epsilon = \pi T(2n + 1)$ and electron valleys $v = \pm\vec{K}$,

$$\begin{aligned} \chi_{AA}^0(\vec{q}, i\omega) &= \chi_{BB}^0(\vec{q}, i\omega) = - \int \frac{d\vec{p}}{(2\pi)^2} \frac{p + p'}{(i\omega)^2 - (p + p')^2}, \\ \chi_{AB}^0(\vec{q}, i\omega) &= \chi_{BA}^0(\vec{q}, i\omega) = \int \frac{d\vec{p}}{(2\pi)^2} \frac{\cos(\phi - \phi')(p + p')}{(i\omega)^2 - (p + p')^2}. \end{aligned} \quad (9)$$

Our results in Eq. 4 and Eq. 9 are in agreement with results of ref. [8], the only difference is that the decomposition of the overlap factor $\cos(\phi - \phi')$ near the Dirac point was performed in Eq. 9. Now by using Eq. 4 and Eq. 9 we should find zeros of Eq. 5, but we didn't obtain zeros or triplet collective excitation for reasonable range of U . In other words, the short-range interaction dose not lead to any neutral collective mode.

ON-SITE INTERACTION: EIGENVECTOR REPRESENTATION

In this section the same problem of RPA in the triplet channel of the on-site interaction will be considered [13], but in the representation of the eigenvectors of the Hamiltonians Eq. 6. For the case of electron valleys \vec{K} and $-\vec{K}$, the eigenvectors are

$$f_s^{\vec{K}} = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{-i\phi/2} \\ se^{i\phi/2} \end{pmatrix}, f_s^{-\vec{K}} = \frac{1}{\sqrt{2}} \begin{pmatrix} -se^{i\phi/2} \\ e^{i\phi/2} \end{pmatrix}, \quad (10)$$

where $f_s^{\pm\vec{K}}$ corresponds to electron states in the conduction band with the energy $v_F p$ and in the valance band with the energy $-v_F p$. The interaction Hamiltonian Eq. 2 in the momentum representation takes this form:

$$\begin{aligned} H_{int} = -\frac{U}{N} \sum_{\vec{p}_1 \vec{p}_2 \vec{q}} \left\{ \Psi_{\vec{p}'_1}^{\uparrow\uparrow} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Psi_{\vec{p}_1}^{\downarrow} \Psi_{\vec{p}'_2}^{\downarrow\uparrow} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Psi_{\vec{p}_2}^{\uparrow} \right. \\ \left. + \Psi_{\vec{p}'_1}^{\uparrow\uparrow} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \Psi_{\vec{p}_1}^{\downarrow} \Psi_{\vec{p}'_2}^{\downarrow\uparrow} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \Psi_{\vec{p}_2}^{\uparrow} \right\}, \end{aligned} \quad (11)$$

where $\vec{p}'_1 = \vec{p}_1 + \vec{q}$, $\vec{p}'_2 = \vec{p}_2 - \vec{q}$, and $\Psi_{\vec{p}}^{\uparrow\downarrow} = (a_{\vec{p}}^{\uparrow\downarrow}, b_{\vec{p}}^{\uparrow\downarrow})^T$ is a two-components column matrix, was made from electron destruction operators. The sum in Eq. 11 over $\vec{p}_1, \vec{p}_2, \vec{q}$ should be distributed among electron valleys. Retaining only the terms corresponding to small \vec{q} (since we search small-momentum collective excitation), we get two intra-valley and two inter-valley terms

$$\begin{aligned} H_{int} = -\frac{U}{N} \sum_{\vec{p}_1 \vec{p}_2 \vec{q}} \sum_{\vec{v}_1, \vec{v}_2} \left\{ \Psi_{\vec{v}'_1}^{\uparrow\uparrow} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Psi_{\vec{v}_1}^{\downarrow} \Psi_{\vec{v}'_2}^{\downarrow\uparrow} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \Psi_{\vec{v}_2}^{\uparrow} \right. \\ \left. + \Psi_{\vec{v}'_1}^{\uparrow\uparrow} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \Psi_{\vec{v}_1}^{\downarrow} \Psi_{\vec{v}'_2}^{\downarrow\uparrow} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \Psi_{\vec{v}_2}^{\uparrow} \right\}, \end{aligned} \quad (12)$$

where $\vec{v}_\alpha = \vec{V}_\alpha + \vec{p}_\alpha$, $\vec{v}'_\alpha = \vec{V}_\alpha + \vec{p}'_\alpha$ and $\vec{V}_\alpha = \pm\vec{K}$. Now by changing the basis of the eigenvectors, the Eq. 10 can be performed according to the relation $\Psi_{\vec{V}+\vec{p}}^{\uparrow\downarrow} = \sum_{s=\pm} f_s^{\vec{V}} c_{\vec{V}+\vec{p},s}^{\uparrow\downarrow}$, where $c_{\vec{V}+\vec{p},s}^{\uparrow\downarrow}$ is the destruction operator for electron from the valley $\vec{V} = \pm\vec{K}$, from the conduction ($s = +1$) or valance ($s = -1$) band, with the momentum \vec{p} (measured from the corresponding Dirac point) and with up/down spin. The result for such transformation of Eq. 12 is,

$$H_{int} = \sum_{\vec{p}_1 \vec{p}_2 \vec{q}} \sum_{\vec{V}_1 \vec{V}_2 = \pm \vec{K}} \sum_{s'_1, s'_2, s_1, s_2 = \pm} \left\{ \Gamma_{\vec{V}_1 \vec{V}_2}^{(1)} + \Gamma_{\vec{V}_1 \vec{V}_2}^{(2)} \right\} \times c_{\vec{V}_1 + \vec{p}'_1, s'_1}^{\uparrow\uparrow} c_{\vec{V}_1 + \vec{p}'_1, s_1}^{\downarrow} c_{\vec{V}_2 + \vec{p}'_2, s'_2}^{\downarrow\uparrow} c_{\vec{V}_2 + \vec{p}'_2, s_2}^{\uparrow} \quad (13)$$

where the vertices are,

$$\begin{aligned} \Gamma_{++}^{(1)} &= \Gamma_{--}^{(2)} = \frac{u}{4} e^{\frac{i}{2}(-\phi_1 + \phi'_1 - \phi_2 + \phi'_2)}, \\ \Gamma_{++}^{(2)} &= \Gamma_{--}^{(1)} = \frac{u}{4} s_1 s'_1 s_2 s'_2 e^{\frac{i}{2}(\phi_1 - \phi'_1 + \phi_2 - \phi'_2)}, \\ \Gamma_{+-}^{(1)} &= \Gamma_{-+}^{(2)} = \frac{u}{4} s_2 s'_2 e^{\frac{i}{2}(-\phi_1 + \phi'_1 + \phi_2 - \phi'_2)}, \\ \Gamma_{+-}^{(2)} &= \Gamma_{-+}^{(1)} = \frac{u}{4} s_1 s'_1 e^{\frac{i}{2}(\phi_1 - \phi'_1 - \phi_2 + \phi'_2)} \end{aligned} \quad (14)$$

where $u = U/N$. The interaction Hamiltonian Eq. 13 gives rise to RPA series of the type depicted in Fig. 2. A vertex in each place of these series can be of type $\Gamma^{(1,2)}$, while electron valleys in the Green function between vertices can be \vec{K} or $-\vec{K}$.

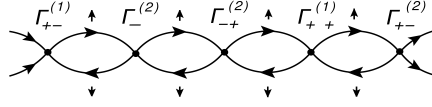


FIG. 2: Each vertex is including $\Gamma^{(1)}$ or $\Gamma^{(2)}$. Each vertex is include interaction between electrons of valance or conduction in each valley. In this chain I considered many types of Γ functions which described interaction of electrons in each valley. This chain is one of the *RPA* chains.

The full vertex can be renormalized by RPA summation, $\Gamma_{\vec{V}_1 \vec{V}_2}^{(i,j)}$, where $i, j = 1, 2$ are the types of left and right vertices in the series. The Bethe-Salpeter equations for this vertex is,

$$\begin{aligned} \Gamma_{\vec{V}_1 \vec{V}_2}^{(ij)}(\vec{p}'_1, s'_1, \vec{p}_1, s_1, \vec{p}'_2, s'_2, \vec{p}_2, s_2; i\omega) &= \\ \delta_{ij} \Gamma_{\vec{V}_1 \vec{V}_2}^{(i)}(\vec{p}'_1, s'_1, \vec{p}_1, s_1, \vec{p}'_2, s'_2, \vec{p}_2, s_2) &= \\ -T \sum_{k \vec{V}} \sum_{\epsilon s s'} \int \frac{d\vec{p}}{(2\pi)^2} \Gamma_{\vec{V}_1 \vec{V}}^{(i)}(\vec{p}'_1, s'_1, \vec{p}_1, s_1, \vec{p}', s', \vec{p}, s) &= \\ \times G_s^{\vec{V}}(\vec{p}, i\epsilon) G_{s'}^{\vec{V}}(\vec{p}', i\epsilon + i\omega) \Gamma_{\vec{V} \vec{V}_2}^{(kj)}(\vec{p}, s, \vec{p}', s', \vec{p}_2, s'_2, \vec{p}_2, s_2; i\omega), & \end{aligned} \quad (15)$$

where $\vec{p}' = \vec{p} + \vec{q}$. The Green functions for eigenstates of the Hamiltonians Eq. 6 at $\mu = 0$ have a simplest form,

$$G_{-}^{\pm \vec{K}}(\vec{p}, i\epsilon) = \frac{1}{i\epsilon - p}, \quad G_{+}^{\pm \vec{K}}(\vec{p}, i\epsilon) = \frac{1}{i\epsilon + p} \quad (16)$$

By further processing, I should note that the vertices Eq. 14 can be decoupled in to *left* and *right* parts,

$$\Gamma_{\vec{V}_1 \vec{V}_2}^{(i)}(\vec{p}'_1, s'_1, \vec{p}_1, s_1, \vec{p}'_2, s'_2, \vec{p}_2, s_2) = \Gamma_{\vec{p}'_1 s'_1 \vec{p}_1 s_1}^{i, \vec{V}_1} \cdot u \cdot \Gamma_{\vec{p}'_2 s'_2 \vec{p}_2 s_2}^{i, \vec{V}_2}, \quad (17)$$

where

$$\gamma_{\vec{p}' s' \vec{p} s}^a = \frac{1}{2} e^{\frac{i}{2}(-\phi + \phi')}, \quad \gamma_{\vec{p}' s' \vec{p} s}^b = \frac{1}{2} s s' e^{\frac{i}{2}(\phi - \phi')}, \quad (18)$$

here the brief notations $1.\vec{K} = 2.(-\vec{K}) = a$ and $2.\vec{K} = 1.(-\vec{K}) = b$ are introduced. Since all bare vertices are decoupled, the summarized vertices also can be decoupled into the left and right parts. The RPA-renormalized interaction is written, $\Gamma_{\vec{V}_1 \vec{V}_2}^{ij} = \Gamma^{i.\vec{V}_1} V_{ij} \Gamma^{i.\vec{V}_2}$. By substituting it into Eq. 15, the interaction equations for this system obtained as,

$$V_{ij} = u + u \sum_{k\vec{V}} \Pi_{ik}^{\vec{V}} V_{kj}, \quad (19)$$

the polarization operators were introduced as,

$$\begin{aligned} \Pi_{ij}^{\vec{V}}(\vec{q}, i\omega) = \\ -T \sum_{\epsilon s s'} \int \frac{d\vec{p}}{(2\pi)^2} \Gamma_{p's'\vec{p}s}^{i.\vec{V}} G_s^{\vec{V}}(\vec{p}, i\epsilon) \Gamma_{\vec{p}s p's'}^{i.\vec{V}} G_{s'}^{\vec{V}}(\vec{p}', i\epsilon + i\omega). \end{aligned} \quad (20)$$

A nontrivial solution for the Eq. 19 occurs when it's determinant is zero,

$$\begin{aligned} \begin{vmatrix} 1 - u\Pi_{11}^{\vec{K}} - u\Pi_{11}^{-\vec{K}} & -u\Pi_{12}^{\vec{K}} - u\Pi_{12}^{-\vec{K}} \\ -u\Pi_{21}^{\vec{K}} - u\Pi_{21}^{-\vec{K}} & 1 - u\Pi_{22}^{\vec{K}} - u\Pi_{22}^{-\vec{K}} \end{vmatrix} = \\ 1 - u(\Pi_{11}^{\vec{K}} + \Pi_{11}^{-\vec{K}} + \Pi_{22}^{\vec{K}} + \Pi_{22}^{-\vec{K}}) \\ + u^2(\Pi_{11}^{\vec{K}} + \Pi_{11}^{-\vec{K}})(\Pi_{22}^{\vec{K}} + \Pi_{22}^{-\vec{K}}) \\ - u^2(\Pi_{12}^{\vec{K}} + \Pi_{12}^{-\vec{K}})(\Pi_{21}^{\vec{K}} + \Pi_{21}^{-\vec{K}}) = 0, \end{aligned} \quad (21)$$

By using Eqs. 16, 17, 18 and Eq. 20, the polarization operators can be easily calculated, by performing summation on $\epsilon = \pi T(2n+1)$ and on $s, s' = \pm 1$, we find

$$\begin{aligned} \Pi_{11}^{\vec{K}} = \Pi_{11}^{-\vec{K}} = \Pi_{22}^{\vec{K}} = \Pi_{22}^{-\vec{K}} &= \frac{1}{2} \int \frac{d\vec{p}}{(2\pi)^2} \frac{p+p'}{(i\omega)^2 - (p+p')^2}, \\ \Pi_{12}^{\vec{K}} = \Pi_{21}^{-\vec{K}} &= -\frac{1}{2} \int \frac{d\vec{p}}{(2\pi)^2} \frac{e^{i(-\phi+\phi')}(p+p')}{(i\omega)^2 - (p+p')^2}, \\ \Pi_{21}^{\vec{K}} = \Pi_{12}^{-\vec{K}} &= -\frac{1}{2} \int \frac{d\vec{p}}{(2\pi)^2} \frac{e^{i(\phi-\phi')}(p+p')}{(i\omega)^2 - (p+p')^2}. \end{aligned} \quad (22)$$

Comparing Eq. 22 with ?? and 4 with 21, we find that $\chi_{AA}^0 = \chi_{BB}^0 = \Pi_{11}^{\vec{K}} + \Pi_{11}^{-\vec{K}} = \Pi_{22}^{\vec{K}} + \Pi_{22}^{-\vec{K}}$ and $\chi_{AB}^0 = \chi_{BA}^0 = \Pi_{12}^{\vec{K}} + \Pi_{12}^{-\vec{K}} = \Pi_{21}^{\vec{K}} + \Pi_{21}^{-\vec{K}}$. Therefore, the sub-lattice representation give the same result as that in the representation of eigenstates in short range interaction. So in this representation we could not find zeros for Eq. 21. Finally we found that the on-site Hubbard interaction can not lead to formation of neutral collective mode. The essential reason is overlap factors which resolve sub-lattices. In the first approach the overlap factors reside in the Green functions, and in the second approach these factors are prescribed to interaction vertices.

LONG RANGE INTERACTION: EFFECTIVE COULOMB INTERACTION

The Coulomb interaction between electrons in graphene is usually considered as long-range, so both sub-lattices participate equally in it, since characteristic range of the interaction is much larger than the lattice constant. In this section we consider long range interaction for finding it's effect on spin-1 collective mode, which was considered in [10]. If the long range (Coulomb) interaction is included interaction between electrons with same and opposite spins on two sub-lattices, we know, it results the plasmon collective mode [11]. The only long range interaction which lead to the Eq. 26 has the form of Eq. 24. Another reason for selecting this form for long range interaction is, the spin-1 collective mode make from interaction between electrons with opposite spins.

If we assume that the Coulomb interaction is represented by constant V between electrons with opposite spin (because the spin-1 mode considered between two electrons with opposite spin), we introduced this interaction Hamiltonian,

$$H_{Coul} = V \sum_{ij} \left\{ a_i^{\uparrow\dagger} a_i^{\uparrow} a_j^{\downarrow\dagger} a_j^{\downarrow} + a_i^{\uparrow\dagger} a_i^{\uparrow} b_j^{\downarrow\dagger} b_j^{\downarrow} + b_i^{\uparrow\dagger} b_i^{\uparrow} a_j^{\downarrow\dagger} a_j^{\downarrow} + b_i^{\uparrow\dagger} b_i^{\uparrow} b_j^{\downarrow\dagger} b_j^{\downarrow} \right\}. \quad (23)$$

In the exchange channel, we have

$$H_{Coul} = -V \sum_{ij} \left\{ a_i^{\uparrow\dagger} a_j^{\downarrow} a_j^{\downarrow\dagger} a_i^{\uparrow} + a_i^{\uparrow\dagger} b_j^{\downarrow} b_j^{\downarrow\dagger} a_i^{\uparrow} + b_i^{\uparrow\dagger} a_j^{\downarrow} a_j^{\downarrow\dagger} b_i^{\uparrow} + b_i^{\uparrow\dagger} b_j^{\downarrow} b_j^{\downarrow\dagger} b_i^{\uparrow} \right\}. \quad (24)$$

If we perform the same calculations starting from Eq. 13, as was done starting from Eq. 13 to derive Eq. 21, we will arrive to the different system,

$$\begin{cases} \chi_{AA} = \chi_{AA}^0 + V\chi_{AA}^0\chi_{AA} + V\chi_{AB}^0\chi_{BA} + V\chi_{AA}^0\chi_{BA} + V\chi_{AB}^0\chi_{BA} \\ \chi_{AB} = \chi_{AB}^0 + V\chi_{AA}^0\chi_{AB} + V\chi_{AB}^0\chi_{BB} + V\chi_{AA}^0\chi_{BB} + V\chi_{AB}^0\chi_{BB} \\ \chi_{BA} = \chi_{BA}^0 + V\chi_{BA}^0\chi_{AA} + V\chi_{BB}^0\chi_{BA} + V\chi_{BA}^0\chi_{BA} + V\chi_{BB}^0\chi_{BA} \\ \chi_{BB} = \chi_{BB}^0 + V\chi_{BA}^0\chi_{AB} + V\chi_{BB}^0\chi_{BB} + V\chi_{BA}^0\chi_{BB} + V\chi_{BB}^0\chi_{BB} \end{cases}. \quad (25)$$

with the solvable condition

$$1 - V(\chi_{AA}^0 + \chi_{AB}^0 + \chi_{BA}^0 + \chi_{BB}^0) = 0, \quad (26)$$

where the susceptibilities are same as Eq. ???. This equation is same as with essential equation of [7] which predicted neutral collective mode in honey comb lattice. If we consider $\chi_{triplet}^0 = \chi_{AA}^0 + \chi_{AB}^0 + \chi_{BA}^0 + \chi_{BB}^0$, the dispersion of the neutral collective mode can be obtained with one of this equations,

$$\Re\chi_{triplet}^0(\vec{q}, \omega) = 1/V \quad \text{or} \quad \Im\chi_{triplet}^0(\vec{q}, \omega) = 0. \quad (27)$$

The zeros of the Eqs. 27 are collections of (\vec{q}, ω) which are plotted in Fig. 3 for $V = 3.2t$. This contour plot displays dispersion of neutral collective mode. The contour plot of $\Re\chi_{triplet}^0$ for $V = 3.2t$ is plotted under the electron-hole ($e - h$) continuum region in $\Gamma \rightarrow K$ direction. But we know this range of U is too large for electrons in graphene. I checked the solution and zeros of Eq. 27 for $0.0t < V < 3.0t$, but I didn't find any zeros. For $V > 3.2t$ we could find dispersions for neutral collective mode near Γ point in $\Gamma - K$ direction which are meaningless. So the long range interaction of type Eq. 23 can not formed the neutral collective mode in undoped graphene without *one - cone approximation*.

The prediction of triplet collective mode in Ref. [7] was due to the *single - cone* approximation in short range on-site interaction that led to scalar form of polarization operator without U^2 terms in Eq. 5. One-cone approximation which lead to elimination of some parts of Eq. 5, is the only reason that predicted of the spin-1 collective mode.

CONCLUSION

In this research the effects of short range Hubbard interaction in two representations and new effective long range interaction Hamiltonian was studied and introduced. The prediction of neutral collective mode in on-site and effective long range interaction is analyzed, and we found that in the RPA method on Hubbard model without single-cone approximation, mixing the overlap functions of two sub-lattices suppressed this collective mode. These two types of interactions have different Hamiltonians, Eq. 13 and Eq. 24, in both I considered interaction between electrons from the electrons with opposite spins. We can't find and predict neutral collective mode as the effect of effective long range and short range interaction because the overlap factors in both case resolve sub-lattices and suppress the neutral collective mode.

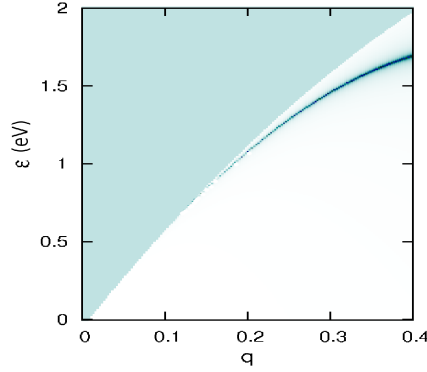


FIG. 3: The contour plot of $\Re \chi_{triplet}^0(\vec{q}, \omega) = 1/V$ is displayed for undoped graphene for $V=3.2t$. The dispersion of spin collective mode appeared below e-h continuum (cyan region) but not in reasonable range of V . The y axis is energy ($\epsilon(eV)$) *vs* momentum (q). The direction in e-h continuum is $\Gamma \rightarrow K$.

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